Sample Question Paper - 29
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours *Maximum Marks : 40*

General Instructions :

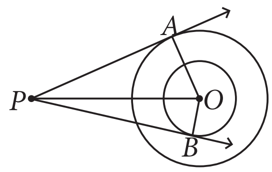
1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. The first and the last terms of an A.P. are 8 and 65 respectively. If the sum of all its terms is 730, find its common difference.

2. Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, formulate the quadratic equation to find the sides of the two squares.

3. In the given figure, the radii of two concentric circles are 7 cm and 8 cm. If PA = 15 cm then find PB.



4. A toy is in the form of a cone mounted on a hemisphere. The diameter of the base of the cone and that of hemisphere is 18 cm and the height of cone is 12 cm. Calculate the surface area of the toy.[Take $\pi = 3.14$]

OR

A solid is in the shape of a cone surmounted by a hemisphere, the radius of each of them being 3.5 cm and the total height of solid is 9.5 cm. Find the volume of the solid.

5. Given that the mean of five number is 27. If one of the number is excluded, the mean gets reduced by 2. Determine the excluded number.

6. A polygon of n sides has $\frac{n(n-3)}{2}$ diagonals. How many sides a polygon has with 54 diagonals?

OR

Find the value(s) of k so that the quadratic equation $3x^2 - 2kx + 12 = 0$ has equal roots.

SECTION - B

7. Find the value of p from the following data, if its mode is 48.

Class-interval	Frequency
0-10	7
10-20	14
20-30	13
30-40	12
40-50	p
50-60	18
60-70	15
70-80	8

8. Draw a line segment of length 6 cm. Using compasses and ruler, find a point P on it which divides it in the ratio 3 : 4.

OR

Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

9. If the mean of the following distribution is 54, find the missing frequency x.

Class	0-20	20-40	40-60	60-80	80-100
Frequency	16	14	24	26	x

10. Two ships are there in the sea on either side of a light house in such a way that the ships and the light house are in the same straight line. The angles of depression of two ships as observed from the top of light house are 60° and 45°. If the height of the light house is 200 m, find the distance between the two ships. [Use $\sqrt{3} = 1.73$]

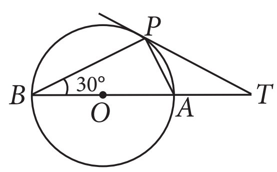
SECTION - C

11. A hemispherical depression is cut out from one face of a cubical block of side 7 cm, such that the diameter of the hemisphere is equal to the edge of the cube. Find the surface area of the remaining solid. $\left[\text{Use } \pi = \frac{22}{7}\right]$

OR

Water is flowing at the rate of 2.52 km/h through a cylindrical pipe into a cylindrical tank, the radius of whose base is 40 cm. If the increase in the level of water in the tank, in half an hour is 3.15 m, find the internal diameter of the pipe.

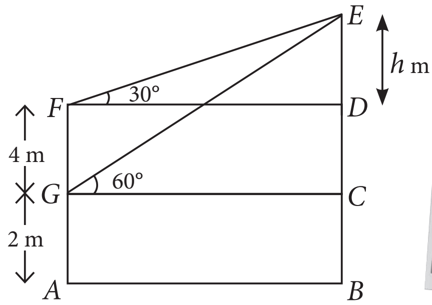
12. In the given figure, O is the centre of the circle and TP is the tangent to the circle from an external point T.

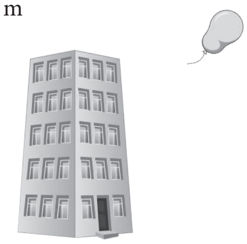


If $\angle PBT = 30°$, prove that $BA : AT = 2 : 1$.

Case Study- 1

13. There are two windows in a house. First window is at the height of 2 m above the ground and other window is 4 m vertically above the lower window. Ankit and Radha are sitting inside the two windows at points G and F respectively. At an instant, the angles of elevation of a balloon from these windows are observed to be 60° and 30° as shown below.





Based on the above information, answer the following questions.

(i) Find the value of h.

(ii) What is the height of the balloon from the ground?

Case Study- 2

14. Meena's mother start a new shoe shop. To display the shoes, she put 3 pairs of shoes in 1st row, 5 pairs in 2nd row, 7 pairs in 3rd row and so on.

On the basis of above information, answer the following questions.

(i) If she puts a total of 120 pairs of shoes, then find the number of rows required.

(ii) What is the difference of pairs of shoes in 17th row and 10th row.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. Let a and d be the first term and common difference respectively of the A.P.

Given, $a = 8$ and $l = 65 = a + (n - 1)d$

$\Rightarrow\ 65 = 8 + (n-1)d \ \Rightarrow\ 57 = (n-1)d$ ...(i)

Also, $S_n = 730$ (Given) $\Rightarrow\ \frac{n}{2}(a + l) = 730$

$\Rightarrow\ n[8 + 65] = 1460 \Rightarrow\ n = \frac{1460}{73} = 20$

Putting value of n in (i), we get $57 = (20 - 1)d$

$\Rightarrow\ 57 = 19d \ \Rightarrow\ d = 3$

2. Let the length of each side of a square be x m. Then, its perimeter is $4x$ m.

It is given that the difference of the perimeters of two squares is 24 m.

$\therefore$ Perimeter of second square $= (24 + 4x)$ m

$\Rightarrow$ Length of each side of second square

$= \frac{24 + 4x}{4}$ m $= (6 + x)$ m

It is given that the sum of the areas of two squares is 468 m^2.

$\therefore\ x^2 + (6 + x)^2 = 468 \Rightarrow x^2 + (36 + 12x + x^2) = 468$

$\Rightarrow\ 2x^2 + 12x - 432 = 0 \Rightarrow x^2 + 6x - 216 = 0$

This is the required quadratic equation.

3. Here, $OA \perp PA$ and $OB \perp PB$

[$\because$ Tangent at any point of a circle is perpendicular to the radius through the point of contact.]

In ΔPAO, $OP^2 = AP^2 + OA^2 = 15^2 + 8^2$

$= 225 + 64 = 289$

$\Rightarrow\ OP = 17$ cm

In ΔPBO, $PB^2 = OP^2 - OB^2 = 17^2 - 7^2 = 289 - 49 = 240$

$\Rightarrow\ PB = \sqrt{240}$ cm $= 4\sqrt{15}$ cm

4. Radius of the base of the cone and hemisphere (r)

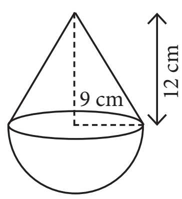
$= \frac{18}{2} = 9$ cm

Height of cone $(h) = 12$ cm

Slant height of cone (l)

$= \sqrt{r^2 + h^2} = \sqrt{9^2 + 12^2}$

$= \sqrt{81 + 144} = \sqrt{225} = 15$ cm



Total surface area of toy

= Curved surface area of hemisphere + Curved surface area of cone

$= 2\pi r^2 + \pi r l = \pi r(2r + l) = 3.14 \times 9(2 \times 9 + 15)$

$= 3.14 \times 9 \times 33 = 932.58$ cm^2

OR

Radius of cone (r) = Radius of hemisphere (r)= 3.5 cm

Total height of solid = 9.5 cm

$\therefore$ Height of cone (h)

$= 9.5 - 3.5 = 6$ cm

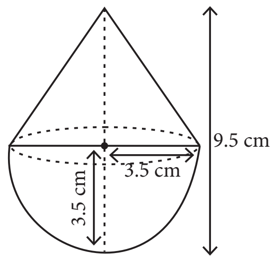
Volume of solid = Volume of cone + Volume of hemisphere

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{\pi r^2}{3}[h + 2r]$

$= \frac{22}{7} \times \frac{3.5 \times 3.5}{3} \times [6 + (2 \times 3.5)]$

$= \frac{22 \times 0.5 \times 3.5 \times 13}{3}$

$= \frac{500.5}{3} = 166.83$ cm^3



5. We have, mean of 5 numbers = 27

Sum of 5 numbers $= 27 \times 5 = 135$

If one number is excluded, then mean of remaining 4 numbers $= 27 - 2 = 25$

Sum of remaining 4 numbers $= 25 \times 4 = 100$

$\therefore$ Excluded number $= 135 - 100 = 35$

6. Given, when number of sides is n, then the number of diagonals is $\frac{n(n-3)}{2}$.

It is given that the number of diagonals = 54

$\Rightarrow\ \frac{n(n-3)}{2} = 54 \Rightarrow n^2 - 3n = 108$

$\Rightarrow\ n^2 - 3n - 108 = 0 \Rightarrow n^2 - 12n + 9n - 108 = 0$

$\Rightarrow\ n(n - 12) + 9(n - 12) = 0 \Rightarrow (n - 12)(n + 9) = 0$

$\Rightarrow\ n = 12$ or $n = -9 \Rightarrow n = 12$

($\because\ n \neq -9$, as number of sides cannot be negative)

$\therefore$ The number of sides of the polygon is 12.

OR

Given, $3x^2 - 2kx + 12 = 0$

Since, for equal roots, $D = 0$

$\Rightarrow\ (-2k)^2 - 4 \times 3 \times 12 = 0$

$\Rightarrow\ 4k^2 - 144 = 0 \Rightarrow k^2 = 36 \Rightarrow k = \pm 6$

7. Here, mode is 48, which lies in the interval 40-50.

$\therefore$ Modal class is 40–50.

So, $l = 40, f_0 = 12, f_1 = p, f_2 = 18, h = 10$

$$\therefore \text{ Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\Rightarrow 48 = 40 + \left(\frac{p-12}{2p-12-18}\right) \times 10 \Rightarrow 8 = \frac{10p-120}{2p-30}$$

$$\Rightarrow 16p - 240 = 10p - 120 \Rightarrow 6p = 120 \Rightarrow p = 20$$

8. Steps of construction :

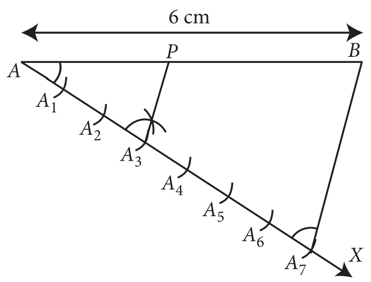
Step-I : Draw a line segment AB of length 6 cm and draw a ray AX making an acute angle with this line segment AB.

Step-II : Locate 7 points, $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ on AX such that $AA_1 = A_1A_2 = A_2A_3$ and so on.

Step-III : Join BA_7.

Step-IV : Through the point A_3, draw a line parallel to BA_7 intersecting AB at point P.

Thus, P is the point that divides line segment AB of length 6 cm in the ratio 3 : 4.



OR

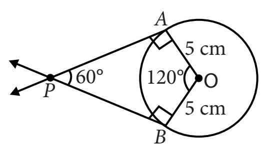
Steps of construction :

Step-I : Draw a circle of radius 5 cm with centre O.

Step-II : At O construct radii OA and OB such that $\angle AOB = 120°$.

Step-III : Draw perpendiculars at A and B such that these perpendiculars intersect at P.

Hence, PA and PB are required tangents.



9.

Class	Class marks (x_i)	Frequency (f_i)	f_ix_i
0-20	10	16	160
20-40	30	14	420
40-60	50	24	1200
60-80	70	26	1820
80-100	90	x	$90x$
		$\Sigma f_i = 80 + x$	$\Sigma f_ix_i = 3600 + 90x$

$$\therefore \text{ Mean} = \frac{\sum f_ix_i}{\sum f_i}$$

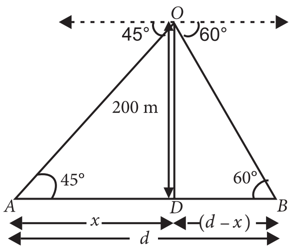
$$\Rightarrow 54 = \frac{3600 + 90x}{80 + x}$$

$$\Rightarrow 54(80 + x) = 3600 + 90x$$

$$\Rightarrow 4320 + 54x = 3600 + 90x$$

$$\Rightarrow 720 = 36x \Rightarrow x = 20$$

10. Let d m be the distance between the two ships. Suppose the distance of one of the ships from the light house is x m, then the distance of the other ship from the light house is $(d - x)$ m.



In right-angled ΔADO, we have

$$\tan 45° = \frac{OD}{AD} \Rightarrow 1 = \frac{200}{x} \Rightarrow x = 200 \quad \text{...(i)}$$

In right-angled ΔBDO, we have

$$\tan 60° = \frac{OD}{BD} \Rightarrow \sqrt{3} = \frac{200}{d - x}$$

$$\Rightarrow (d - x)\sqrt{3} = 200 \Rightarrow \sqrt{3}d - x\sqrt{3} = 200$$

$$\Rightarrow \sqrt{3}d - 200\sqrt{3} = 200 \quad \text{(Using (i))}$$

$$\Rightarrow \sqrt{3}d = 200(\sqrt{3} + 1)$$

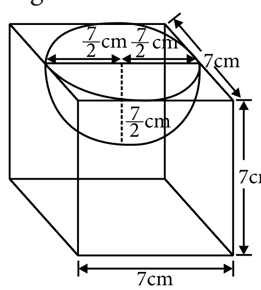
$$\Rightarrow d = \frac{200(\sqrt{3}+1)}{\sqrt{3}} = \frac{200 \times 2.73}{1.73} = 315.60$$

Thus, the distance between two ships is approximately 315.60 m.

11. Diameter of hemisphere = Edge of cube = 7 cm

Radius of hemisphere

$(r) = \frac{7}{2}$ cm



Required surface area = surface area of cube – area of top of hemisphere + curved surface area of hemisphere

$= 6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$

$$= 6(7)^2 + \pi\left(\frac{7}{2}\right)^2 = 6 \times 49 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$= 294 + 38.5 = 332.5 \text{ cm}^2$

OR

Rate of flow of water = 2.52 km/h

$= \frac{2.52 \times 1000 \times 100}{60 \times 60}$ cm/s = 70 cm/s

In 1s, water flows = 70 cm

In 30 min, water flows = 70 × 30 × 60 cm

= 126000 cm = h_1 (say)

Let internal radius of pipe = r_1

Height of water level in the tank in half an hour,

$h_2 = 3.15 \times 100 = 315$ cm

Radius of tank $r_2 = 40$ cm

Volume of water flows from the cylindrical pipe in half an hour = Volume of water in cylindrical tank

$\Rightarrow \pi r_1^2 h_1 = \pi r_2^2 h_2$

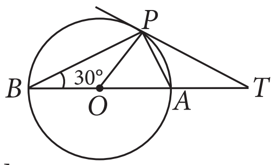
$\Rightarrow r_1^2 \times 126000 = 40 \times 40 \times 315$

$\Rightarrow r_1^2 = \frac{40 \times 40 \times 315}{126000} = 4 \Rightarrow r_1 = 2\text{ cm}$

∴ Internal diameter of pipe = $2r_1$ = 4 cm

12. Given : O is the centre of the circle and TP is the tangent to circle and $\angle PBT = 30°$.

To prove : $\frac{BA}{AT} = \frac{2}{1}$



Construction : Join OP.

Proof : $\angle BPA = 90°$ [Angle in a semicircle]

In ΔBPA, $\angle BPA + \angle PBA + \angle BAP = 180°$ [By angle sum property]

$\Rightarrow 90° + 30° + \angle BAP = 180°$

$\Rightarrow \angle BAP = 60°$

Also, $\angle OPT = 90°$ [∵ PT is a tangent to the circle]

And $OP = OA$ [Radii of same circle] ...(i)

∴ $\angle OAP = \angle OPA = 60°$ ...(ii)

$\Rightarrow \angle APT = 90° - 60° = 30°$

Now, $\angle OAP + \angle PAT = 180°$ [Linear pair]

$\Rightarrow 60° + \angle PAT = 180°$ [Using (ii)]

$\Rightarrow \angle PAT = 120°$

In ΔPAT, $\angle PAT + \angle APT + \angle PTA = 180°$ [By angle sum property]

$\Rightarrow 120° + 30° + \angle PTA = 180°$

$\Rightarrow \angle PTA = 30°$

Now, $\angle APT = \angle PTA = 30°$

$\Rightarrow PA = AT$ ...(iii)

Also in ΔOAP, $\angle AOP = 180° - 60° - 60° = 60°$

∴ $\angle AOP = \angle OPA$

$\Rightarrow PA = OA$ ...(iv)

Hence, $PA = AT = OA = OP$ [Using (i), (iii) and (iv)]

Now, $BA = BO + OA = 2\,OA$ [∵ $OA = OB$]

$\Rightarrow BA = 2AT$ [∵ $OA = AT$]

$\Rightarrow \frac{BA}{AT} = \frac{2}{1}$

13. (i) In ΔEFD, $\tan 30° = \frac{ED}{DF}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{DF}$

$\Rightarrow DF = h\sqrt{3}$ m

Now, in ΔGCE,

$\tan 60° = \frac{EC}{GC} = \frac{h+4}{DF}$

$\Rightarrow \sqrt{3} = \frac{h+4}{\sqrt{3}h} \Rightarrow 3h = h + 4 \Rightarrow h = 2$

(ii) Height of the balloon from the ground = BE

$= BC + CD + DE = 2 + 4 + 2 = 8$ m

14. Number of pairs of shoes in 1st, 2nd, 3rd row, ... are 3, 5, 7, ...

So, it forms an A.P. with first term $a = 3$, $d = 5 - 3 = 2$

(i) Let n be the number of rows required.

∴ $S_n = 120$

$\Rightarrow \frac{n}{2}[2(3) + (n-1)2] = 120$

$\Rightarrow n^2 + 2n - 120 = 0 \Rightarrow n^2 + 12n - 10n - 120 = 0$

$\Rightarrow (n + 12)(n - 10) = 0 \Rightarrow n = 10$

So, 10 rows required to put 120 pairs.

(ii) No. of pairs in 17th row = $t_{17} = 3 + 16(2) = 35$

No. of pairs in 10th row = $t_{10} = 3 + 9(2) = 21$

∴ Required difference = 35 – 21 = 14